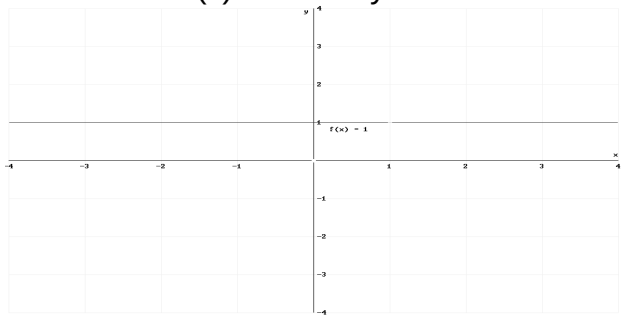
# Funktionen-Katalog



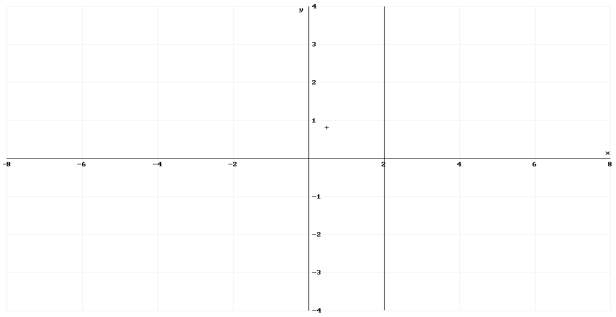
## *I. Geraden*

f(x) = 1 oder y = 1



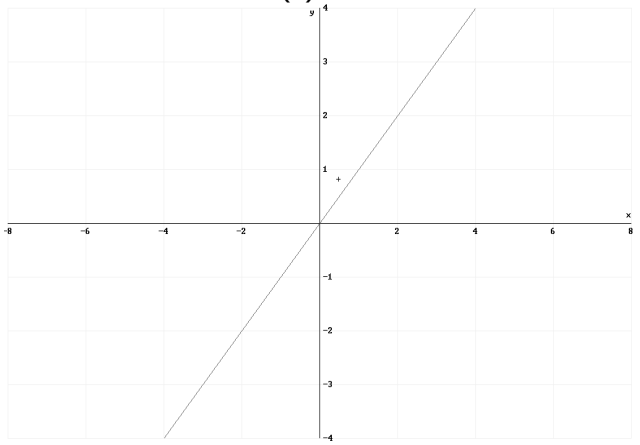
eine Gerade parallel zur x-Achse

x = 1



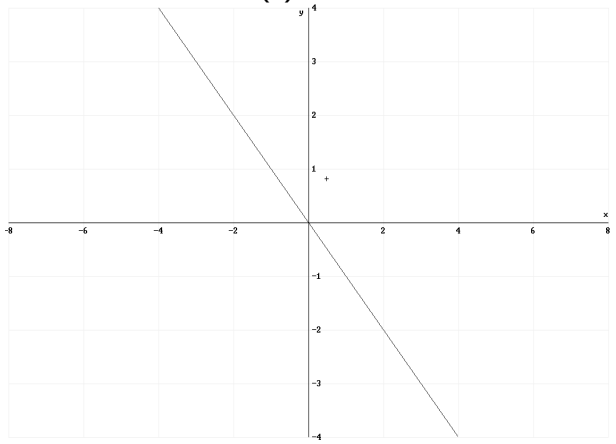
Gerade parallel zur y- Achse (keine Funktion)

f(x) = x



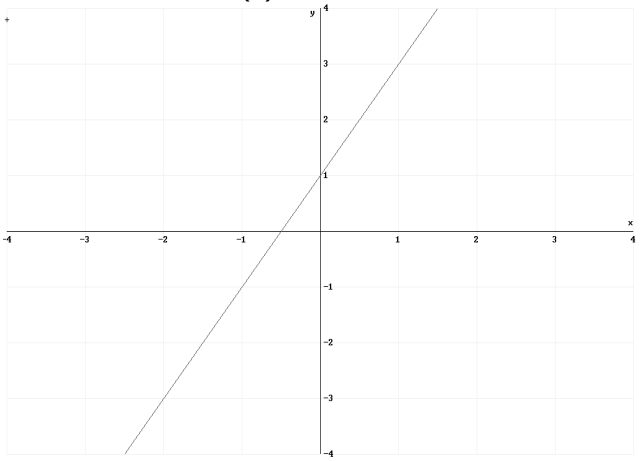
Erste Winkelhalbierende - Steigung 1

f(x) = - x



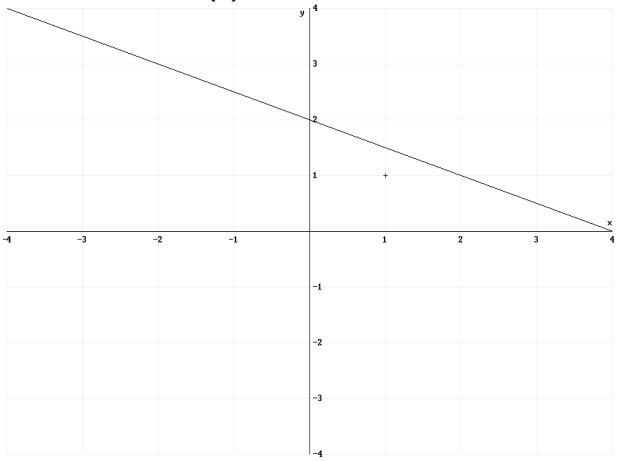
Zweite Winkelhalbierende - Steigung -1

f(x) = 2 x + 1



Gerade mit Steigung 2 und y-Achsenabschnitt 1

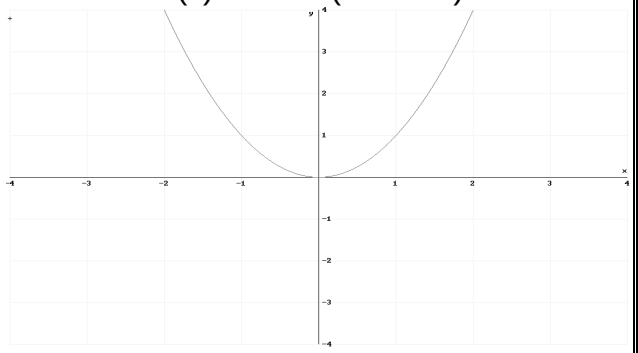
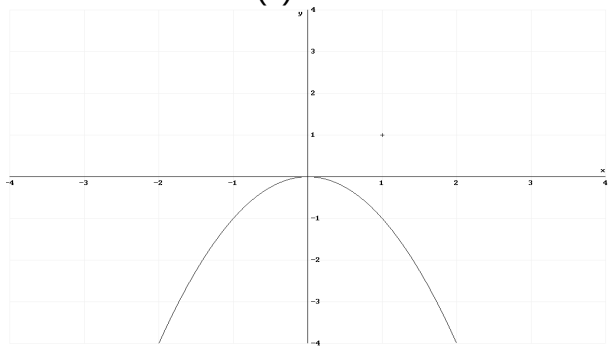
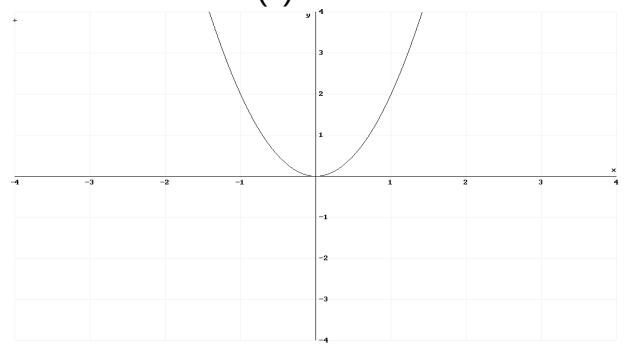
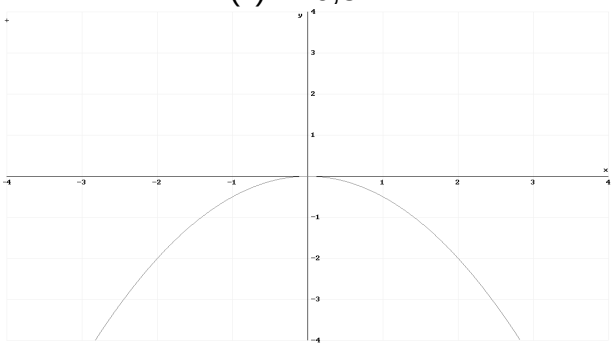
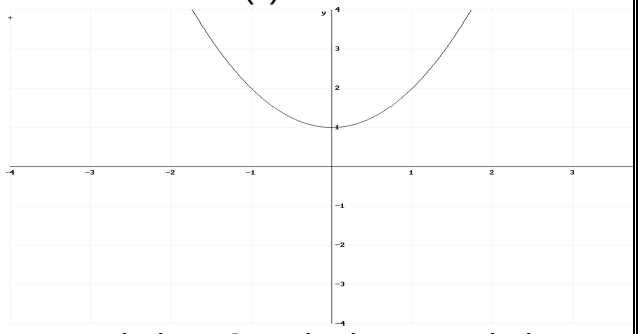
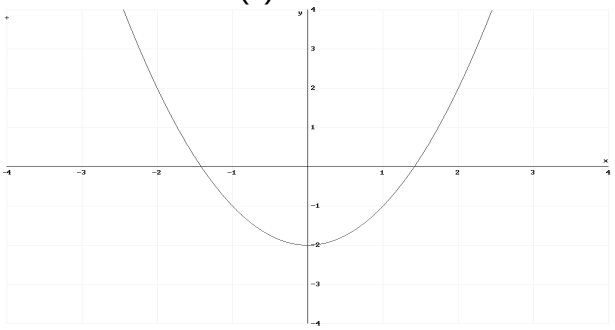
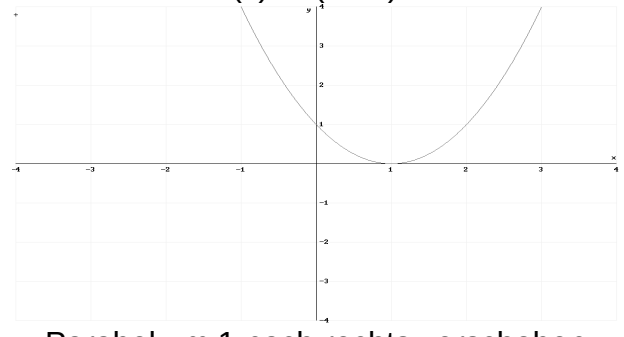
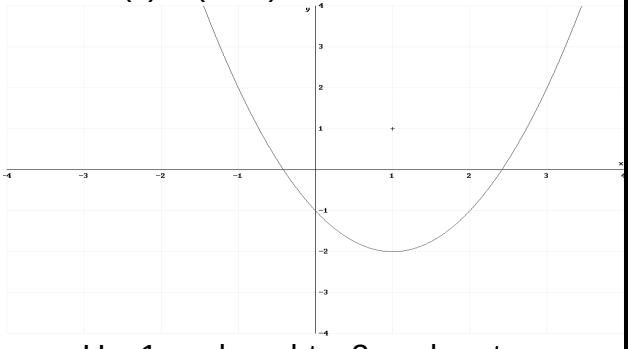
f(x) = - 0,5 x + 2



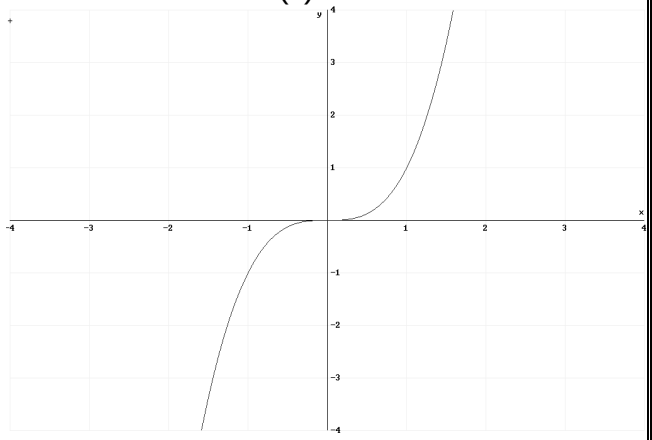
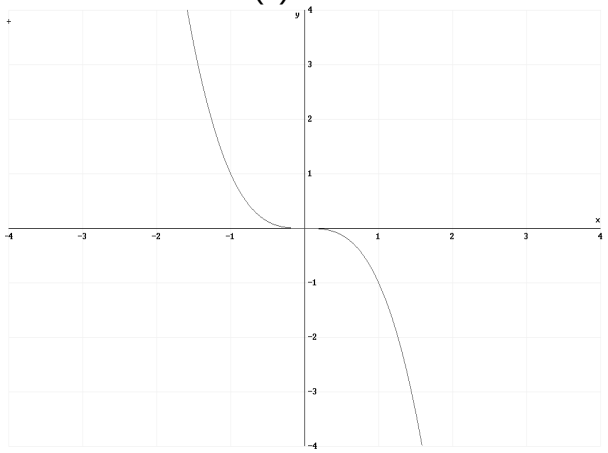
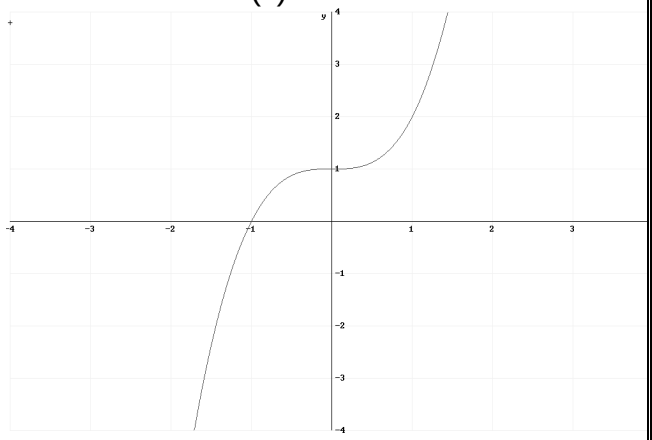
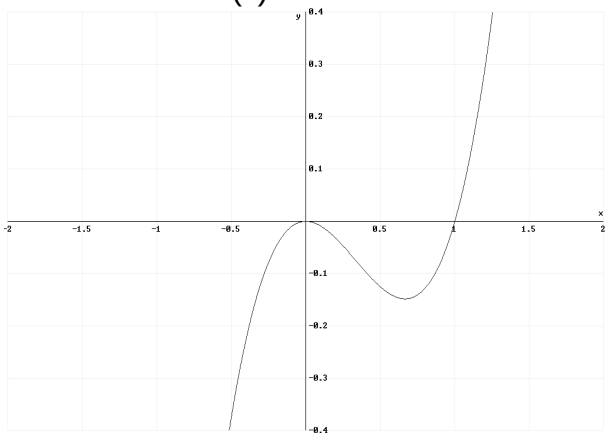
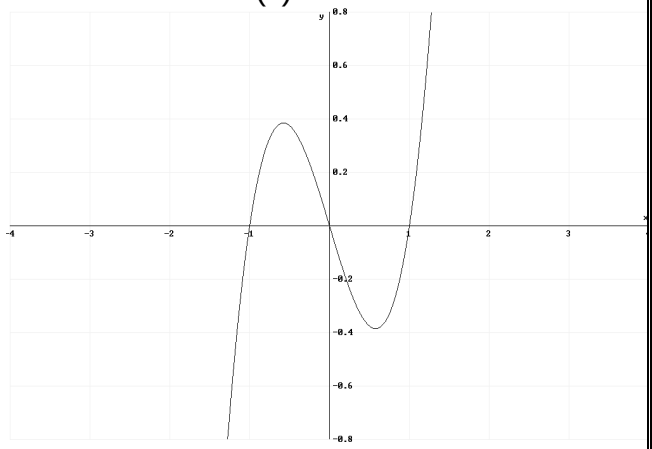
Gerade mit Steigung -0,5 (nach unten) und y-Achsenabschnitt 2

## *Ganzrationale Funktionen*

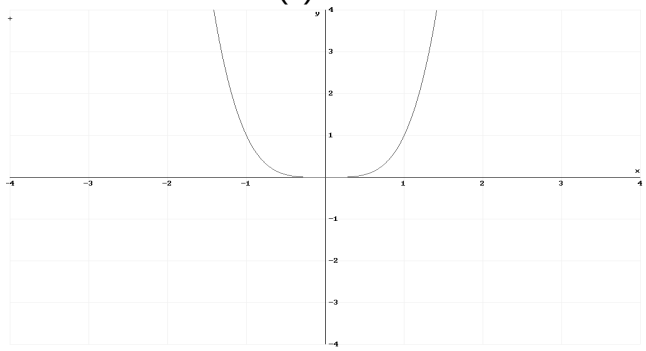
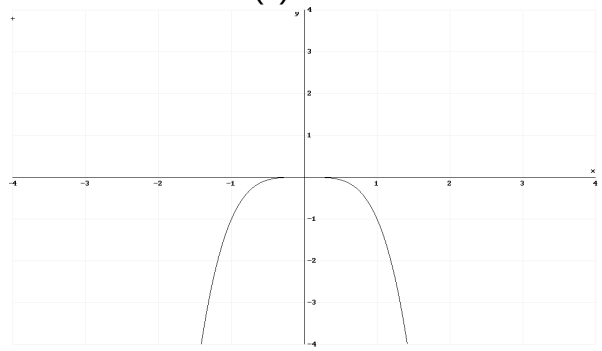
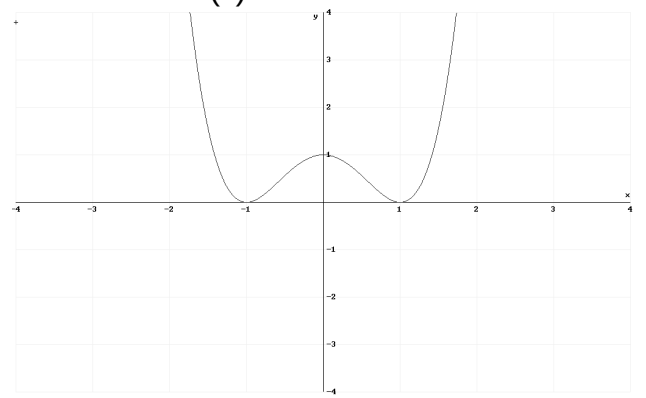
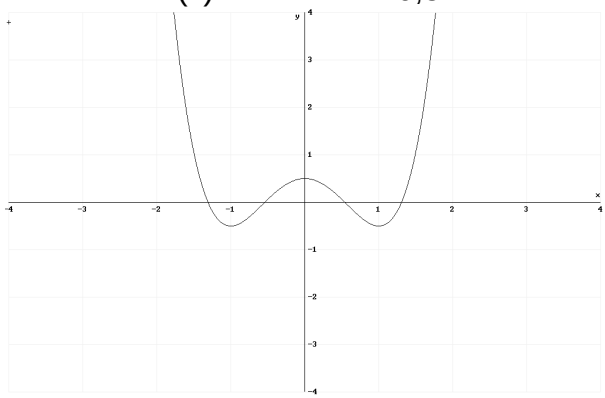
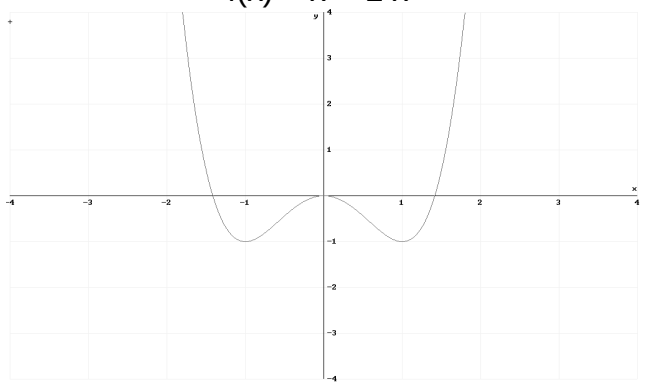
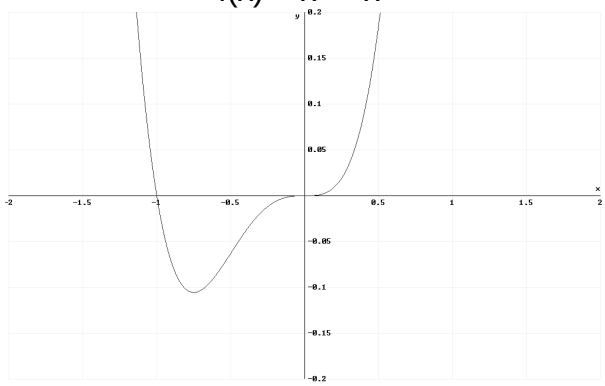
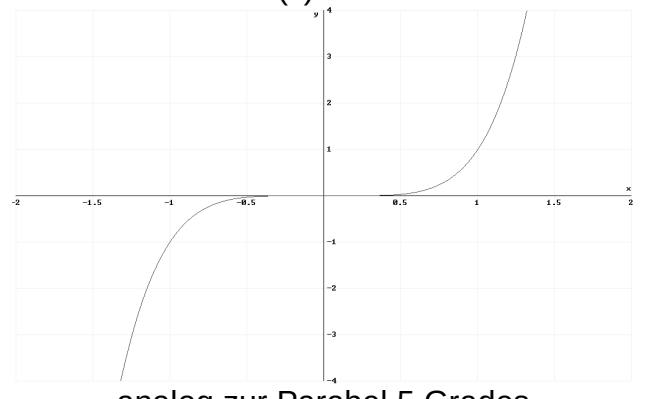
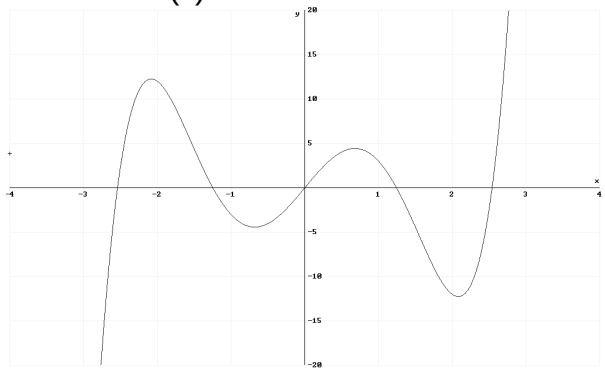
1.) Parabeln 2-ten Grades

| | |
|---|---|
| $f(x) = x^2$ (Parabel)<br>  <br>Normalparabel - 1 Tiefpunkt - achsensymmetrisch | $f(x) = -x^2$<br>  <br>an der x-Achse gespiegelt - 1 Hochpunkt |
| $f(x) = 2\,x^2$<br>  <br>Steilere Parabel (Faktor 2) | $f(x) = -0{,}5\,x^2$<br>  <br>Parabel umgeklappt / flacher (Faktor 0,5)<br>1 Hochpunkt |
| $f(x) = x^2 + 1$<br>  <br>Parabel um 2 nach oben verschoben<br>keine Nullstelle | $f(x) = x^2 - 2$<br>  <br>Parabel um 2 nach unten verschoben<br>2 Nullstellen |
| $f(x) = (x - 1)^2$<br>  <br>Parabel um 1 nach rechts verschoben | $f(x) = (x - 1)^2 - 2 = x^2 - 2x - 1$<br>  <br>Um 1 nach rechts, 2 nach unten verschoben |

## 2.) Parabeln 3-ten Grades

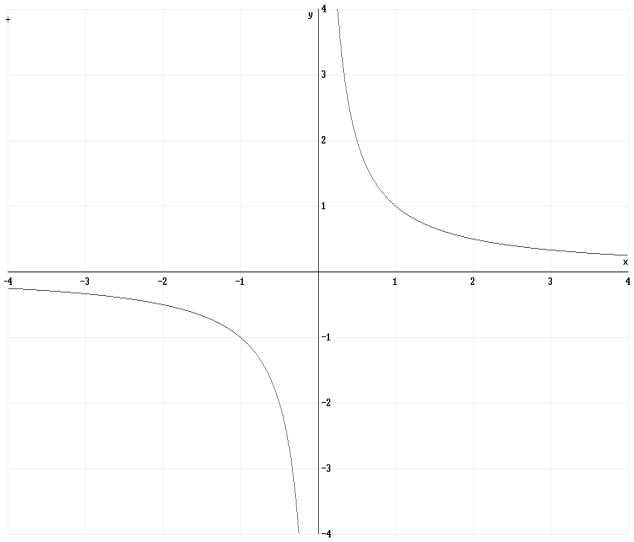
| | |
|---|---|
| $f(x) = x^3$<br>  <br>Parabel 3-ten Grades - punktsymmetrisch<br>1 Wendepunkt - 1 Nse - monoton wachsend | $f(x) = -x^3$<br>  <br>monoton fallend |
| $f(x) = x^3 + 1$<br>  <br>um 1 nach oben verschoben - 1 Nullstelle | $f(x) = x^3 - x^2$<br>  <br>HP berührt x-Achse - TP - WP - 2 Nullstellen |
| $f(x) = x^3 - x$<br>  <br>3 Nullstellen -HP - TP - WP | |

3.) Parabeln höheren Grades

| | |
|---|---|
| $f(x) = x^4$  analog zur Parabel 2-ten Grades - aber steiler | $f(x) = -x^4$  an der x-Achse gespiegelt |
| $f(x) = x^4 - 2\,x^2 + 1$  2 Tpe - 1 HP - 2 Wpe - 2 Nse | $f(x) = x^4 - 2\,x^2 + 0{,}5$  4 Nullstellen |
| $f(x) = x^4 - 2\,x^2$  3 Nullstellen | $f(x) = x^4 + x^3$  1 TP - 2 Wpe - 2 Nsen |
| $f(x) = x^5$  analog zur Parabel 5 Grades | $f(x) = x^5 - 8\,x^3 + 10\,x$  4 Nsen - 2 Hpe - 2 Tpe - 3 Wpe |

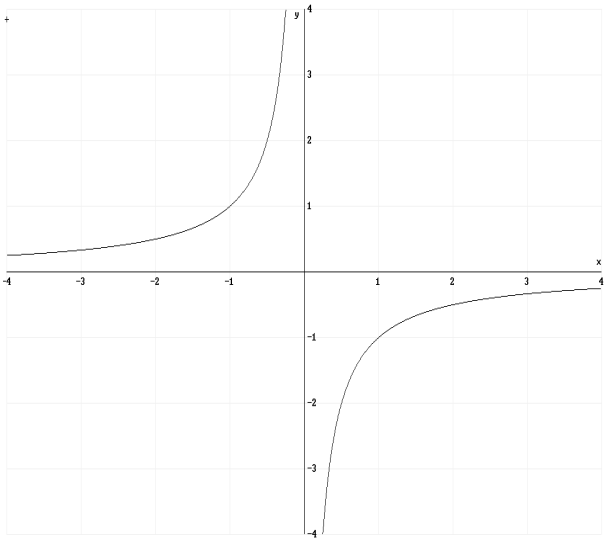
## *II. Gebrochenrationale Funktionen*

f(x) = $\frac{1}{x}$ (Hyperbel)



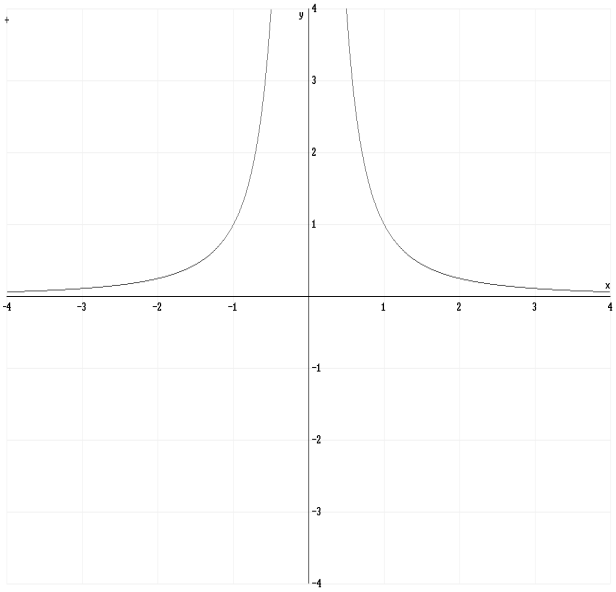
waagrechte Asymptote y = 0 - senkrechte Asymptote x = 0 mit Vorzeichenwechsel

f(x) = -$\frac{1}{x}$



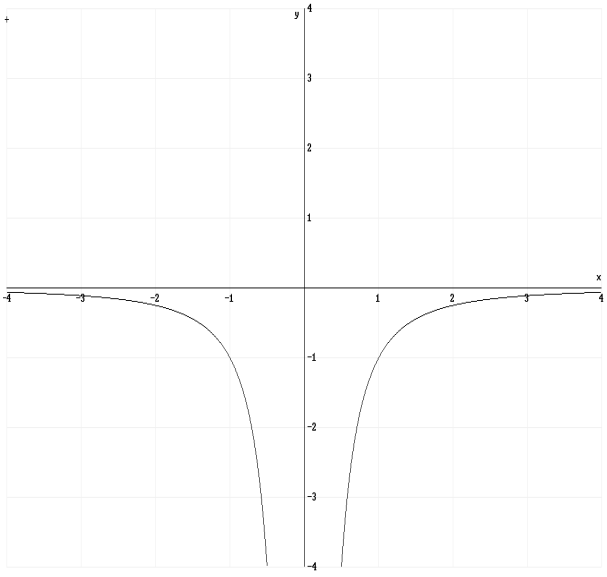
an der x-Achse gespiegelt

f(x) = $\frac{1}{x^2}$



ohne Vorzeichenwechsel

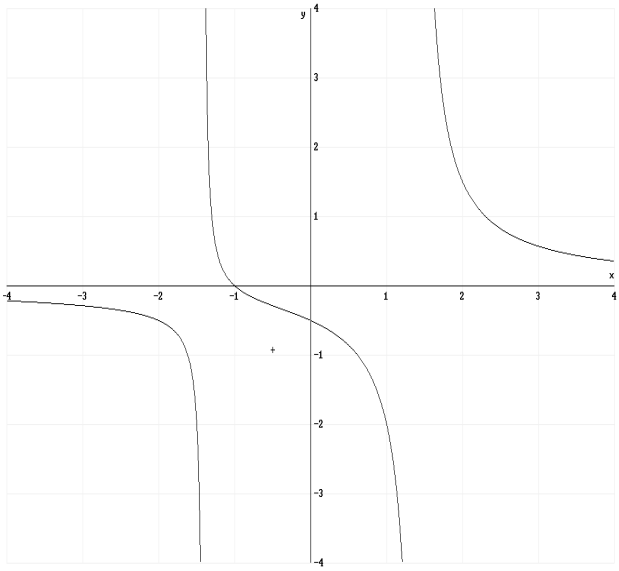
f(x) = -$\frac{1}{x^2}$



an der x-Achse gespiegelt

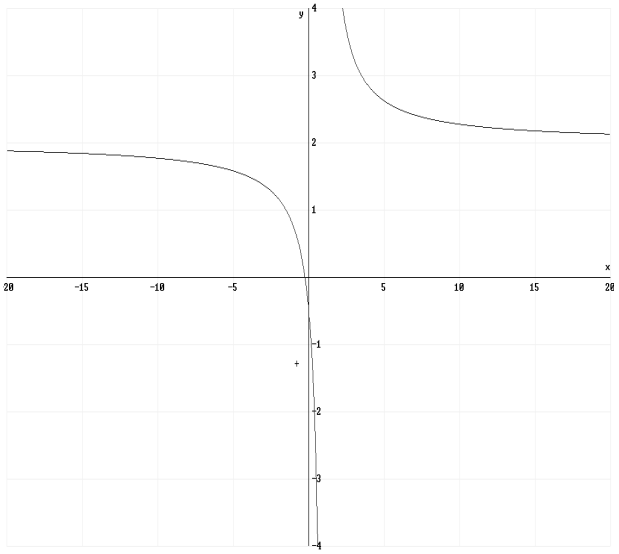
Verschiedene Verhalten für $x \to \pm\infty$
(Maßgeblich sind Zählergrad und Nennergrad - es gibt vier Fälle)

f(x) = $\frac{x^2+x}{x^3-2x}$



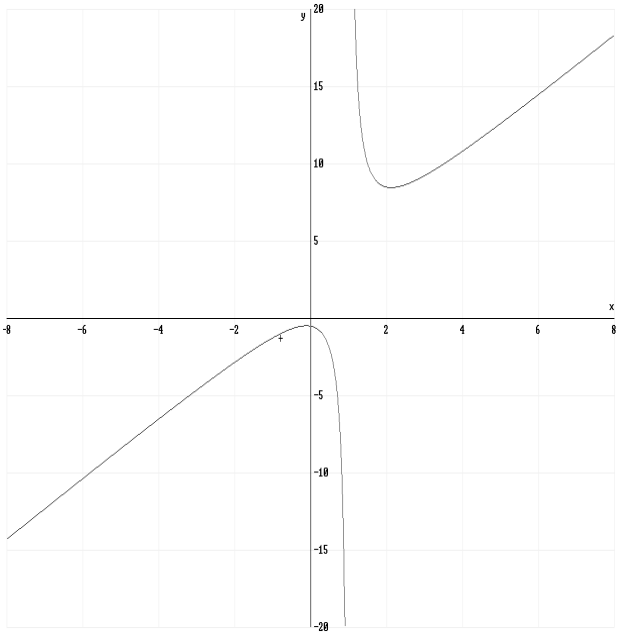
waagrechte Asymptote y = 0,
da Zählergrad < Nennergrad

f(x) = $\frac{4x^2+x}{2x^2-2x}$



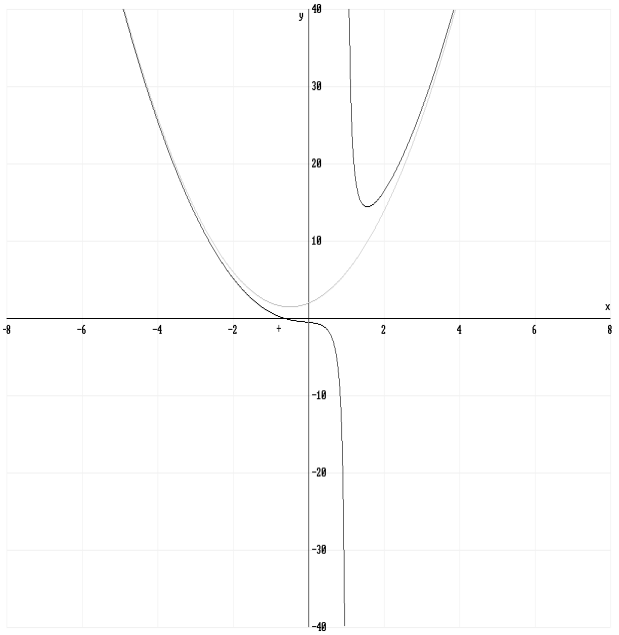
waagrechte Asymptote y = 4/2 = 2
(Koeffizienten vor höchsten Exponenten dividieren), da Zählergrad = Nennergrad

f(x) = $\frac{4x^3+x}{2x^2-2x} = 2x+2+\frac{5}{2(x-2)}$



Schiefe Asymptote y = 2 x + 2
(nach Polynomdivision),
da Zählergrad = Nennergrad +1

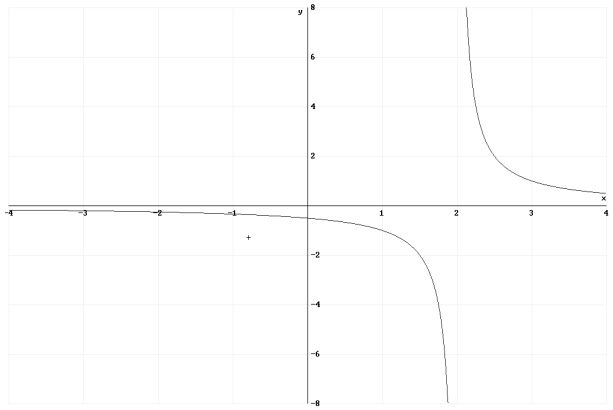
f(x) = $\frac{4x^4+x}{2x^2-2x} = 2x^2+2x+2+\frac{5}{2(x-2)}$



Näherungspolynom y = 2x² + 2x + 2
(nach Polynomdivision),
da Zählergrad > Nennergrad +1

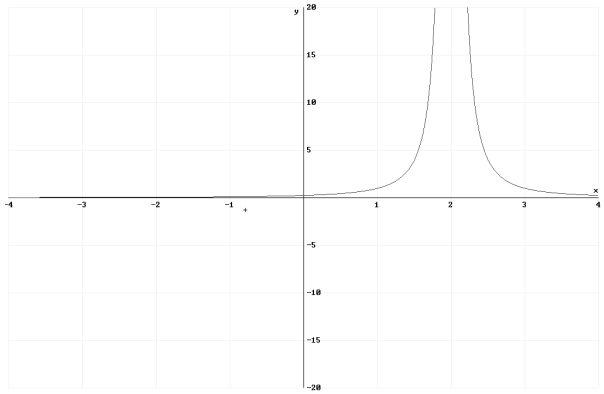
Die senkrechten Asymptoten (= Polstellen)
Entscheidend ist, den Term zu faktorisieren und zu kürzen, um so die Nullstellen von Nenner und Zähler bestimmen zu können - es gibt drei Fälle: Polstelle mit und ohne Vorzeichenwechsel, Lücke

f(x) = $\frac{1}{x-2}$



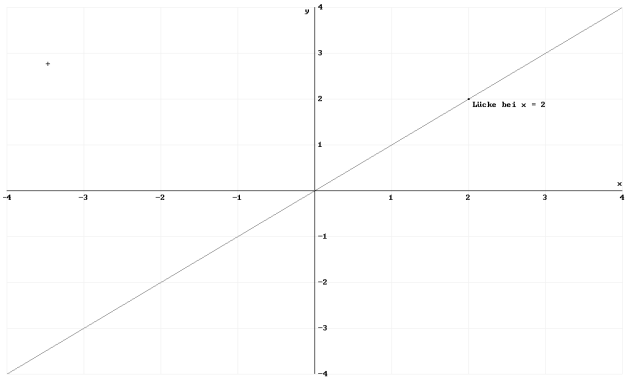
senkrechte Asymptote bei x = 2,
da 2 Nullstelle des Nenners -
mit Vorzeichenwechsel, da einfache Nse

f(x) = $\frac{1}{(x-2)^2}$



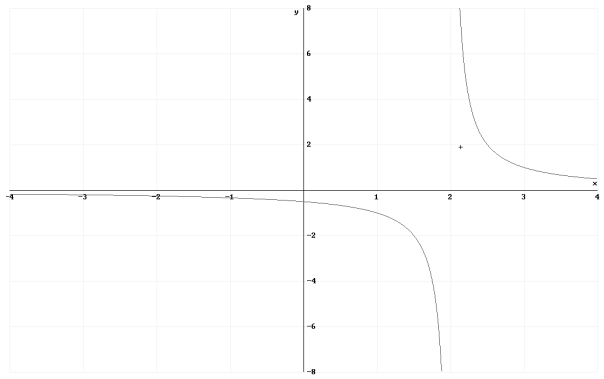
senkrechte Asymptote bei x = 2,
da 2 Nullstelle des Nenners -
ohne Vorzeichenwechsel, da doppelte Nse

f(x) = $\frac{x(x-2)}{x-2} = x$



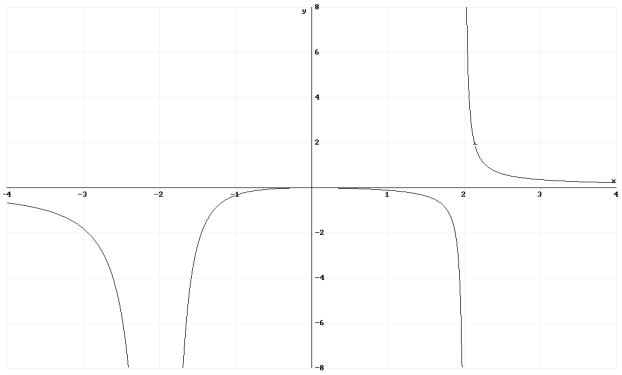

Lücke bei x = 2 - keine Polstelle, da die Nullstelle des Nenners sich herauskürzt

f(x) = $\frac{x(x-2)}{x(x-2)^2} = \frac{x}{x-2}$



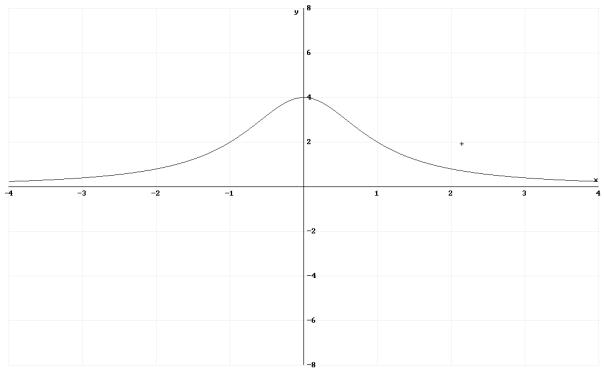
Lücke bei x = 0 (herausgekürzt) und Polstelle mit VZW bei x = 2 (einfache Nullstelle nach dem kürzen)

f(x) = $\frac{x^2(x+1)(x-2)}{x(x+1)(x-2)^2(x+2)^2} = \frac{x}{(x-2)(x+2)^2}$



Lücken bei x = 0 und x = -1 (herausgekürzt), Polstelle mit VZW bei x = 2 und Polstelle ohne VZW bei x = 2

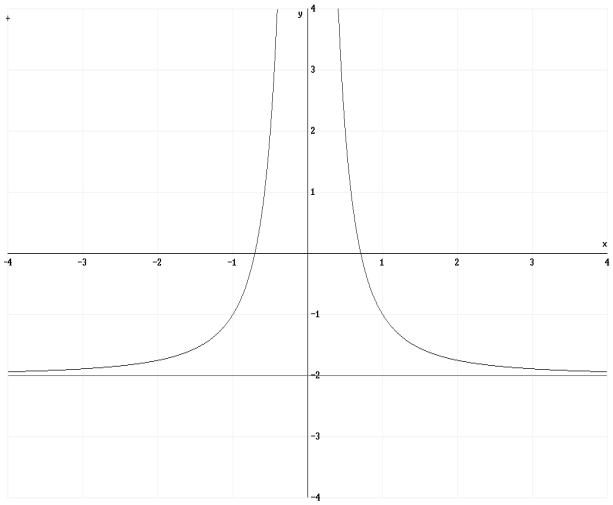
f(x) = $\frac{4}{x^2+1}$



keine Polstelle oder Lücke, da der Nenner nie Null wird

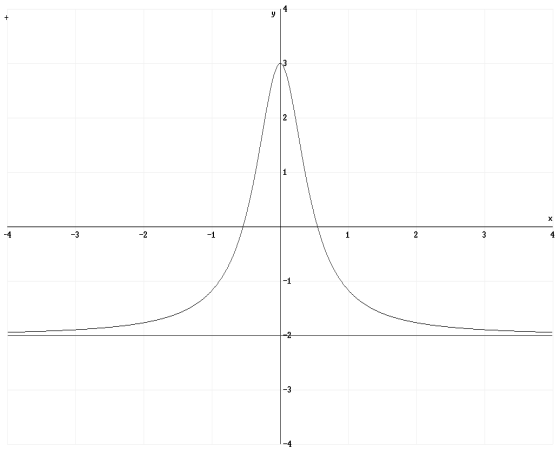
Weitere Effekte:
Nullstellen, Extrempunkte, Wendepunkte, Asymptote wird geschnitten

f(x) = $\frac{1}{x^2} - 2$



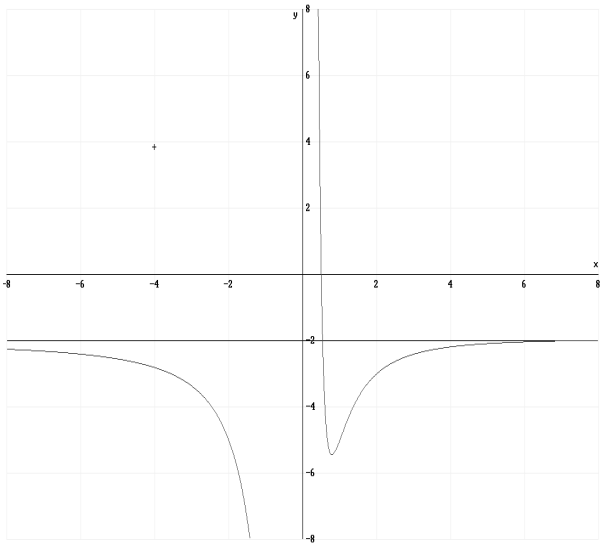
waagrechte Asymptote y = -2
2 Nsem

f(x) = $\frac{1}{x^2 + 0{,}2} - 2$



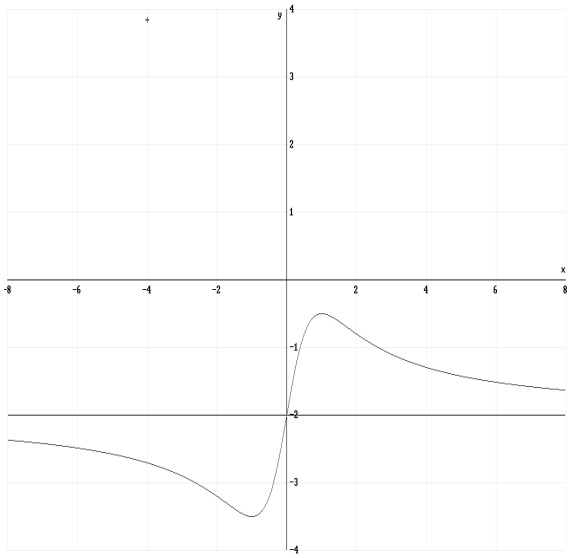
waagr. Asy.: y = -2 - keine senkr. Asy
2 Nsen - 1 HP - 2 Wpe

f(x) = $\frac{x^2 - 8x + 4}{x^3} - 2$



waagr. Asy.: y = -2 - senkr. Asy. x = 0 -
1 TP - 1 WP - f(x) schneidet waagr. Asy

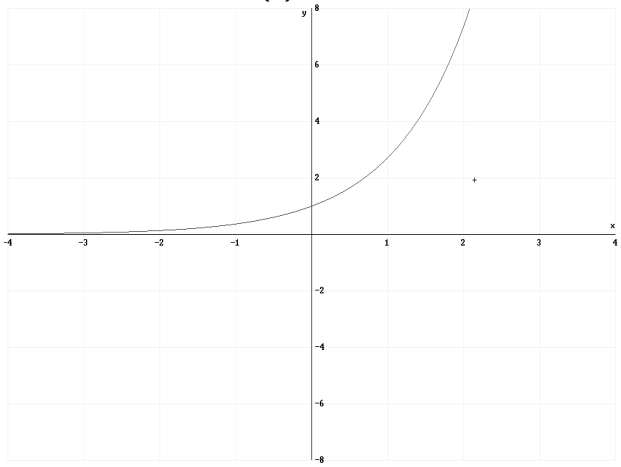
f(x) = $\frac{3x}{x^2 + 1} - 2$



waagr. Asy.: y = -2 - keine senkr. Asy. -
1 HP - 1 TP - 1 WP - f(x) schneidet
waagr. Asy.

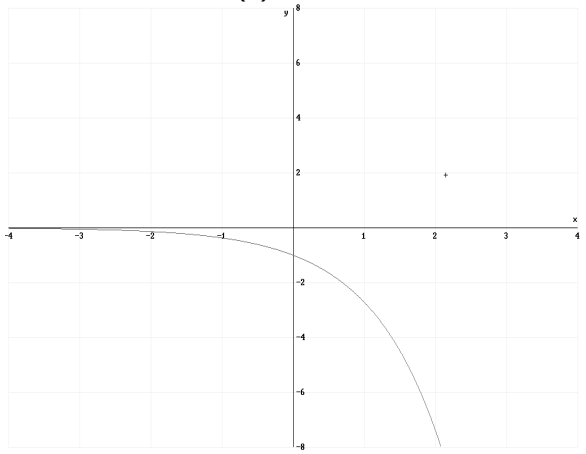
## *IV. Exponentialfunktionen*

f(x) = $e^x$



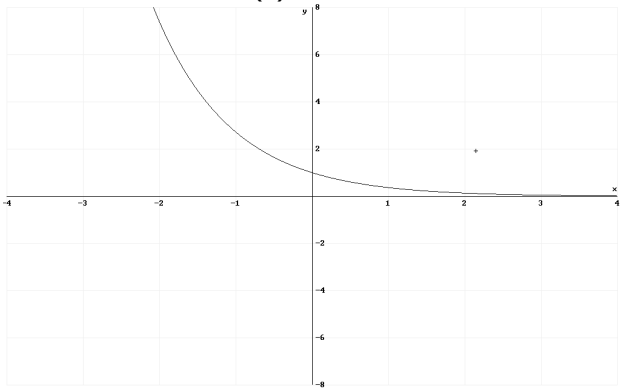
monoton wachsend - Asymptote y = 0
immer positiv

f(x) = $-e^x$



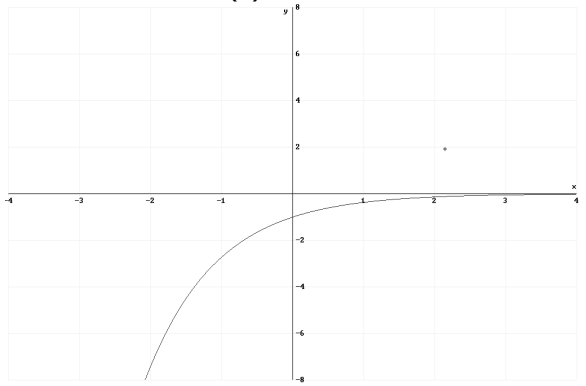
monoton fallend - Asymptote y = 0
immer negativ

f(x) = $e^{-x}$



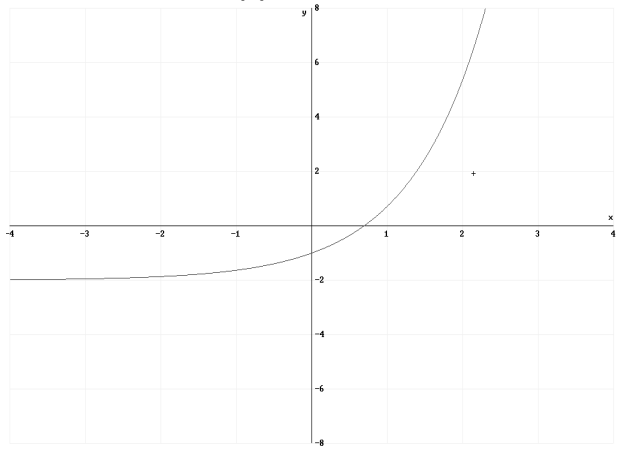
monoton fallend - Asymptote y = 0
immer positiv

f(x) = $-e^{-x}$



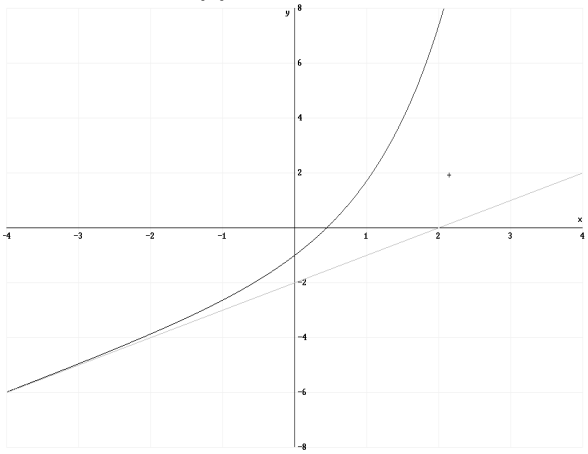
monoton wachsend - Asymptote y = 0
immer negativ

f(x) = $e^x - 2$



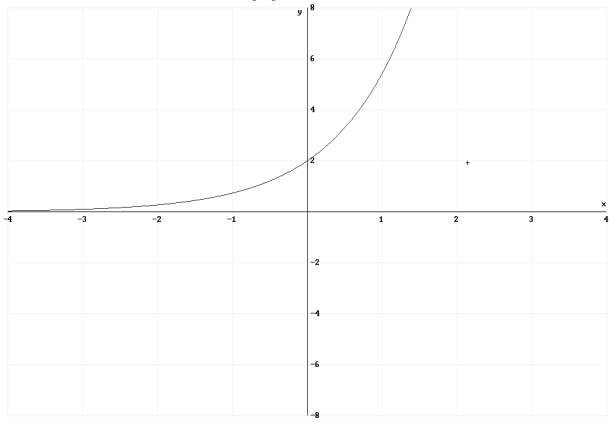
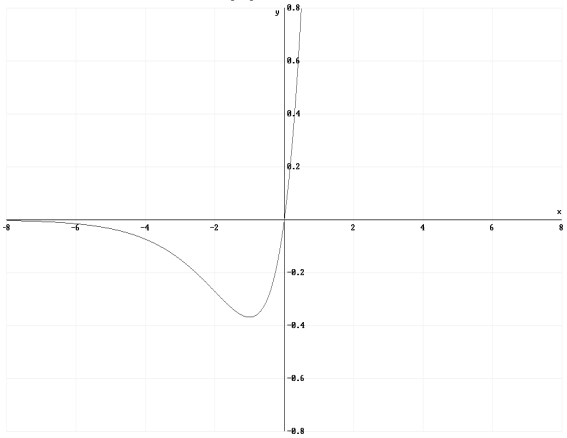
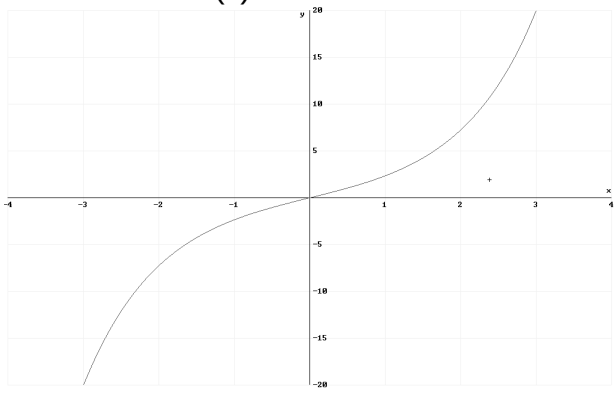
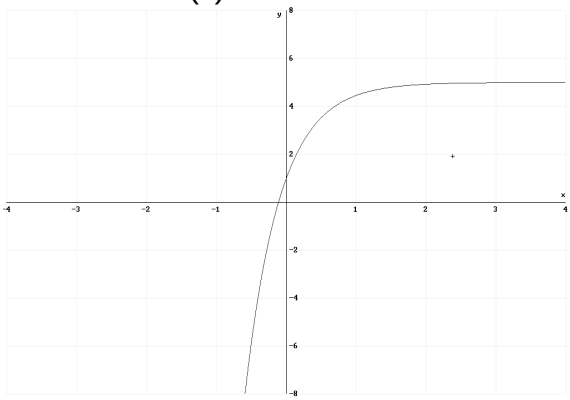
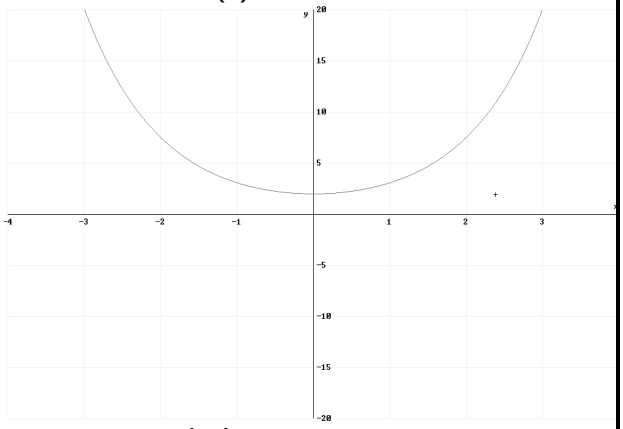
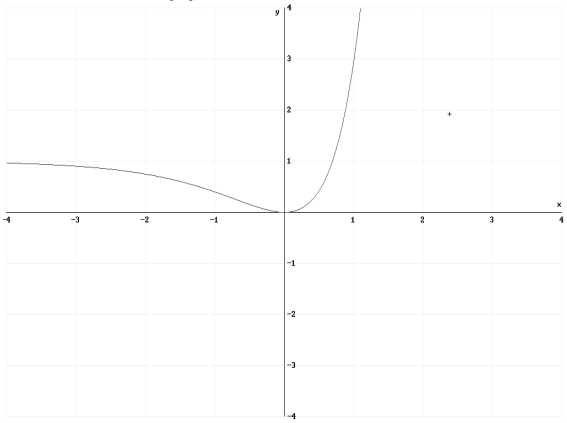
Asymptote y = -2 (nach unten verschoben)
Nullstelle bei x = ln2

f(x) = $e^x + x - 2$



schiefe Asymptote y = x -2
Nullstelle (mit Newtonverfahren) x = 0,44...

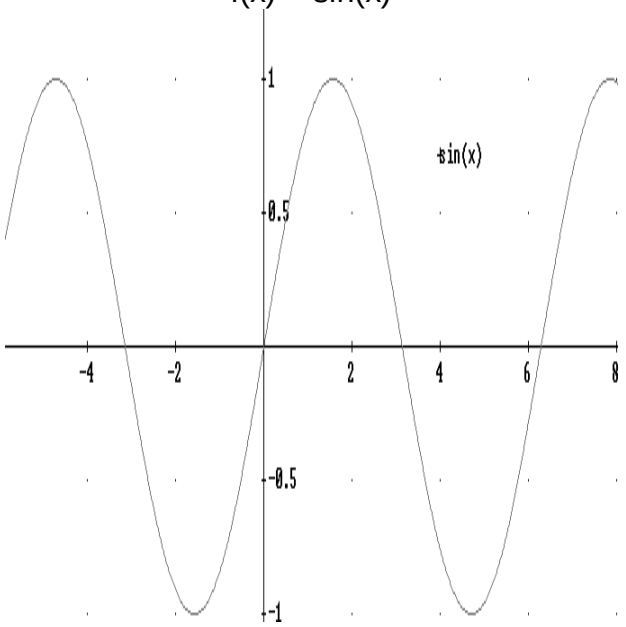
Besonderheiten

| | |
|---|---|
| f(x) = $2e^x$  y-Achsenabschnitt bei 2 | f(x) = $xe^x$  Asymptote bei y= 0 („$e^x$ fällt schneller“)<br>Nullstelle (Faktor!) bei x = 0<br>zusätzlich TP und WP |
| f(x) = $e^x - e^{-x}$  keine Asymptote | f(x) = $5 - 4e^{-2x}$  Begrenztes Wachstum (monoton wachsend, Asymptote bei y = 5) |
| f(x) = $e^x + e^{-x}$  keine Asymptote | f(x) = $e^{2x} - 2e^x + 1$  Asymptote y = 1<br>Nullstelle (über Substitution: $e^x$ = u): x = 0 |

## *VII. Trigonometrische Funktionen*

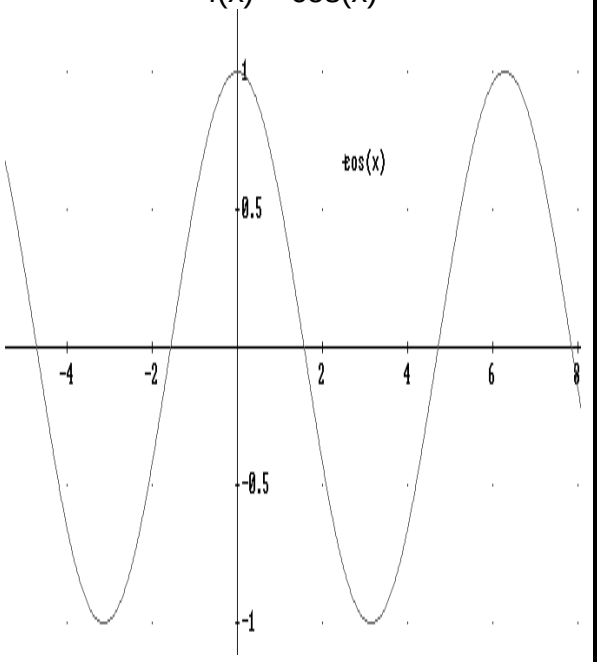
Alles im Bogenmaß ( Taschenrechner: RAD statt DEG - Es gilt 180° $\hat{=}\,\pi$ - Umrechnung: $x = \frac{\pi}{180°}\alpha$ )

f(x) = sin(x)



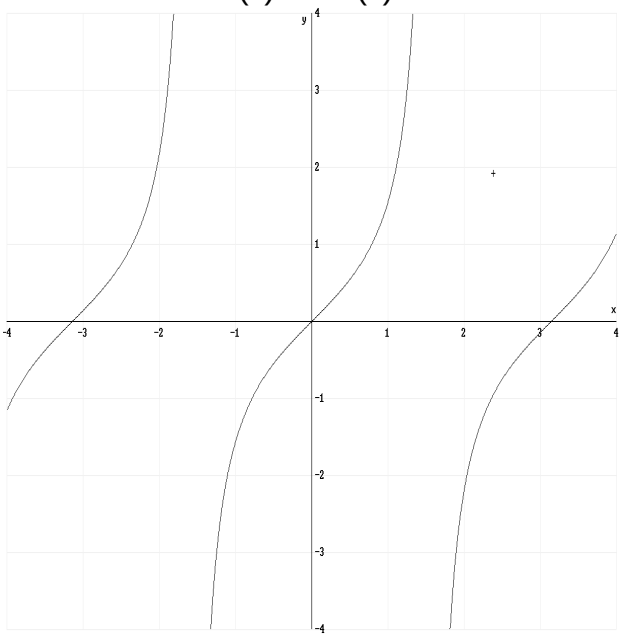

Periode 2π
Amplitude 1 (also von -1 bis +1)
durch (0|0) mit Steigung 1

f(x) = cos(x)



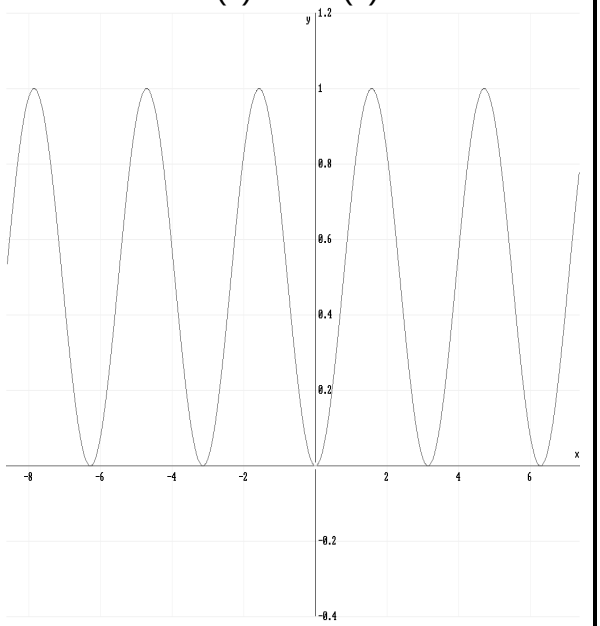

Periode 2π
Amplitude 1 (also von -1 bis +1)
durch (0|1) mit Steigung 0

f(x) = tan(x)




Periode π/2
Polstellen bei -π/2, + π/2, …

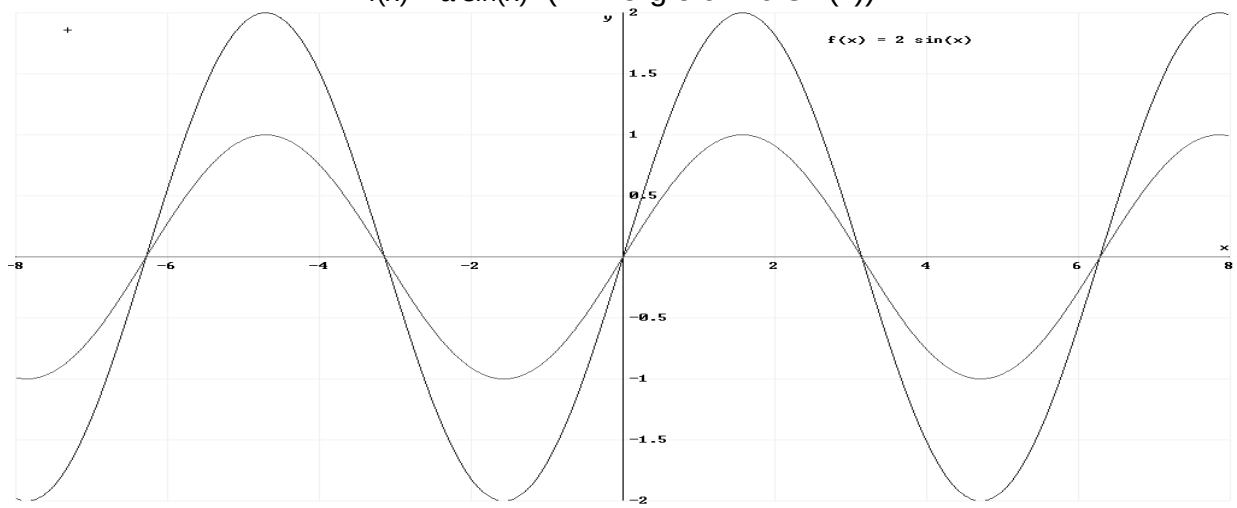
f(x) = sin²(x)




Periode π/2
Zwischen 0 und 1

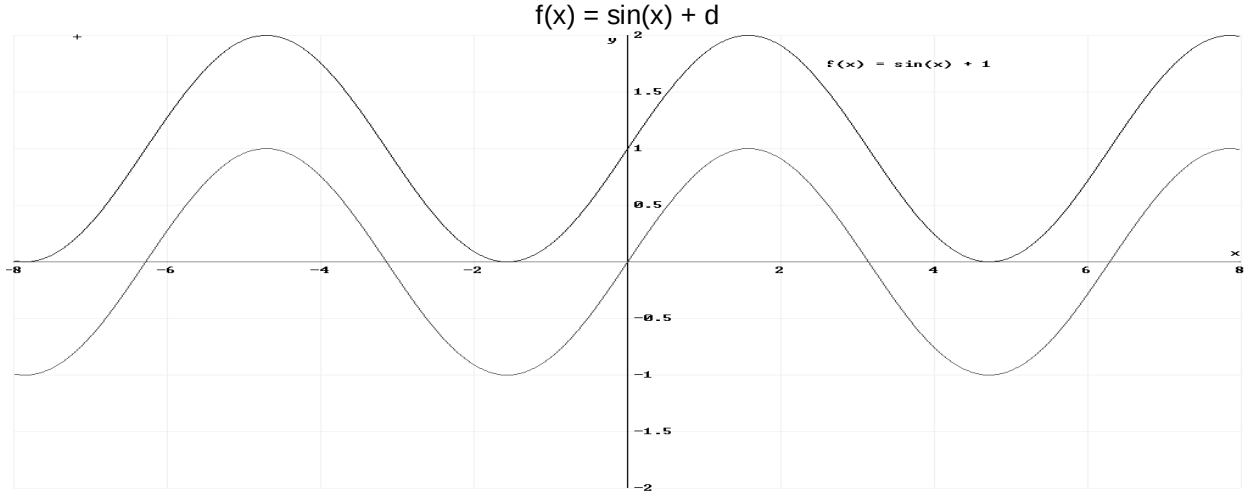
## Allgemeine Sinusfunktion f(x) = a sin(bx + c) + d

f(x) = a sin(x) (im Vergleich zu sin(x))



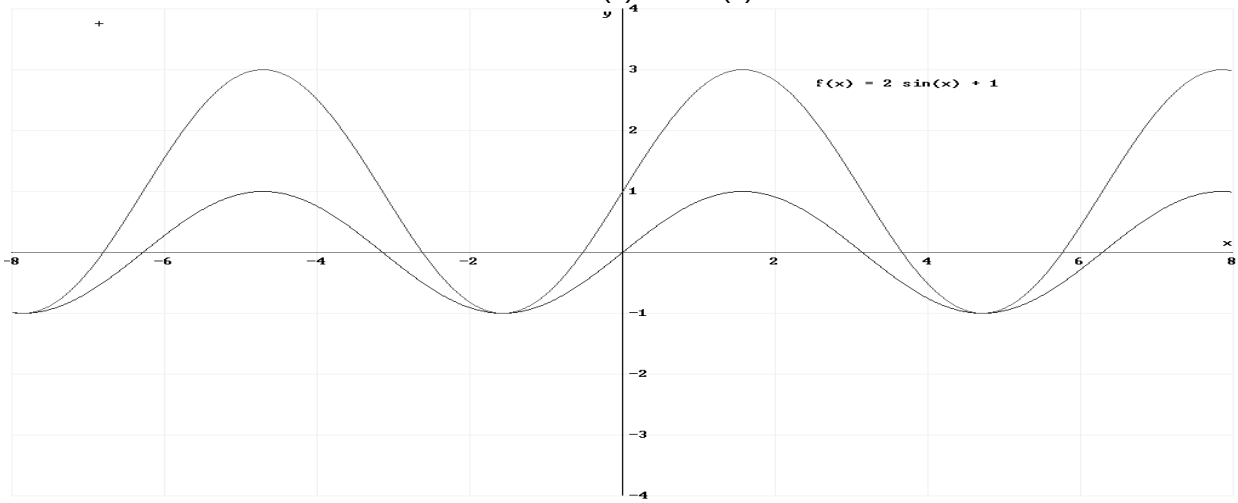

Streckung in y-Richtung um den Faktor a
Dieser Faktor vor der Sinusfunktion heißt Amplitude. Er bestimmt den "Ausschlag" der Sinusfunktion

f(x) = sin(x) + d



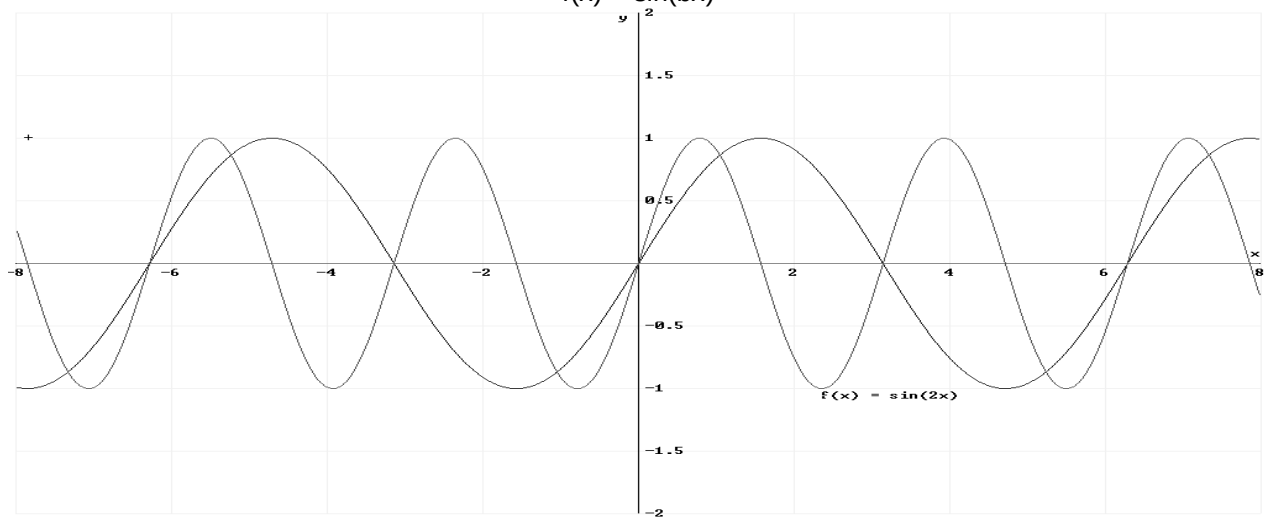

Verschiebung in y-Richtung um d:

Zusammen: f(x) = a sin(x) + d



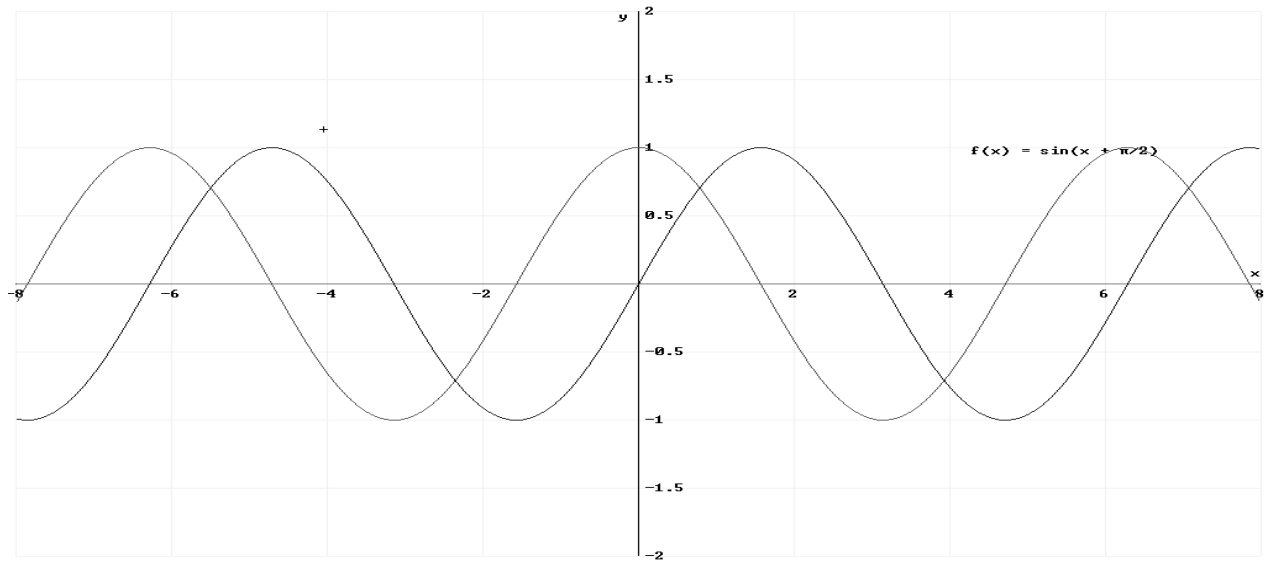

Streckung und Verschiebung in y-Richtung: Funktion oszilliert zwischen -a +d und a + d .

f(x) = sin(bx)



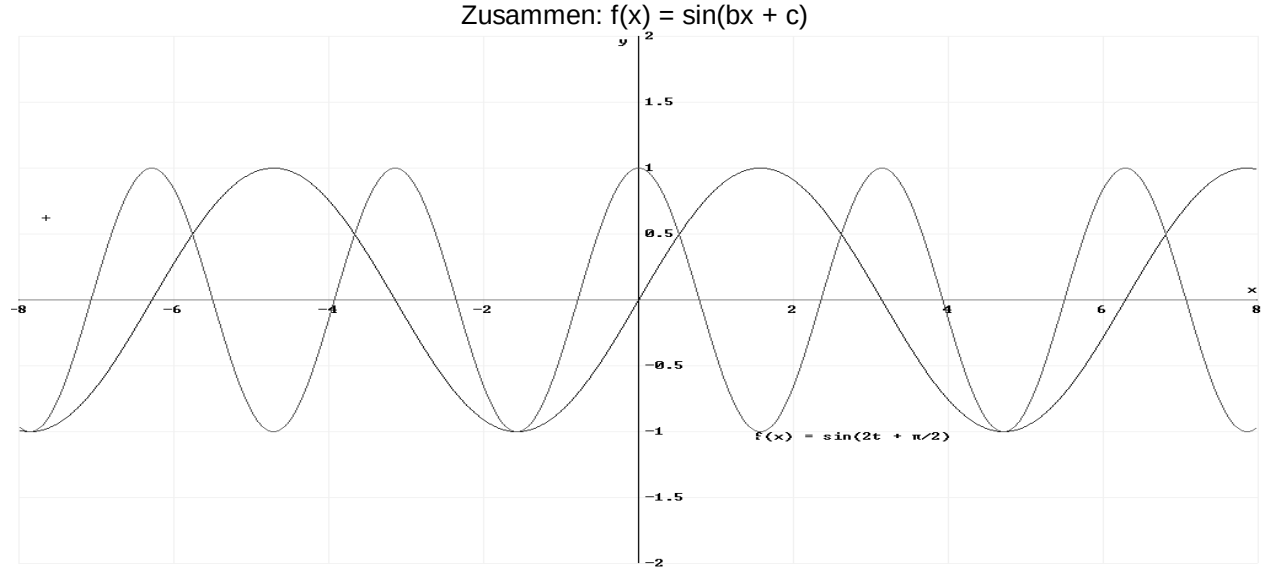

Stauchung in x-Richtung um den Faktor b (Periode T = 2 π /b):

f(x) = sin(x + c)




Verschiebung in x-Richtung nach links um d: Nullpunkt: x + c = 0 π x = - c

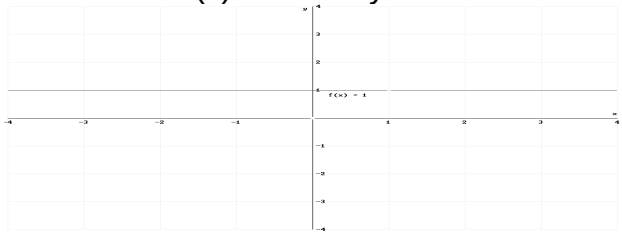
Zusammen: f(x) = sin(bx + c)



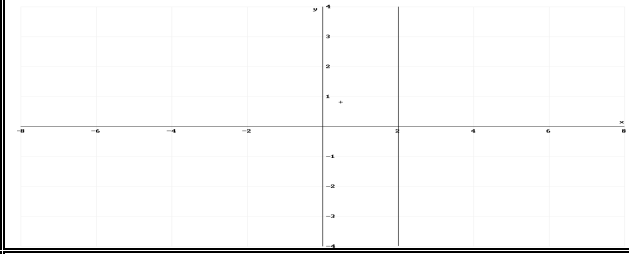

Periode T = 2π/b, Verschiebung nach links um c/b: Nullpunkt: bx + c π x = - c/b

## *VI. Die wichtigsten Schaubilder im Überblick*

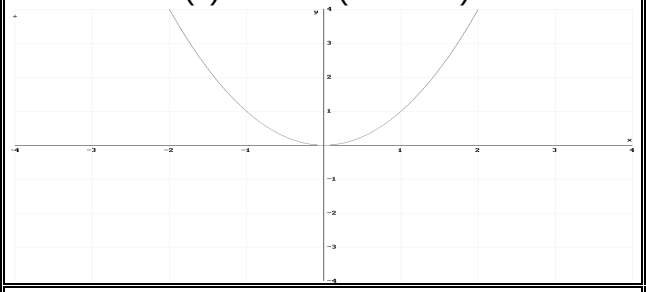
f(x) = 1 oder y = 1



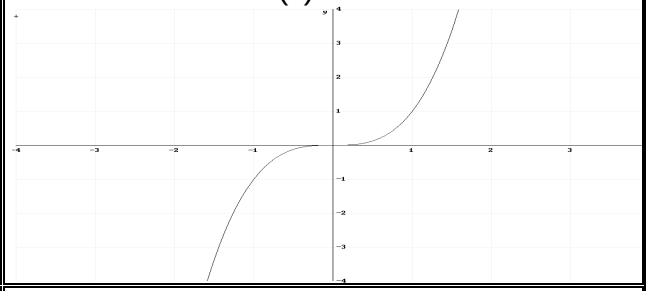
x = 1



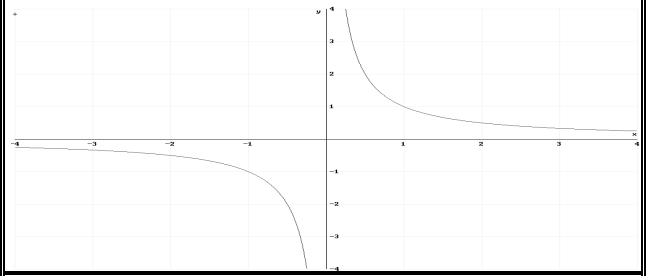
f(x) = x² (Parabel)



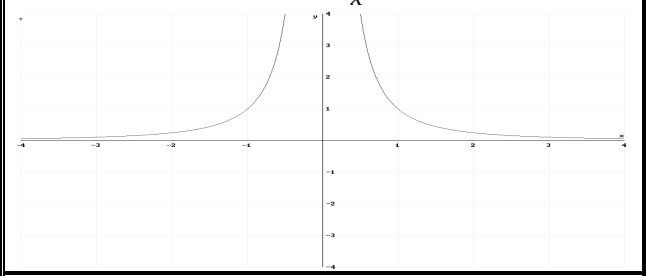
f(x) = x³



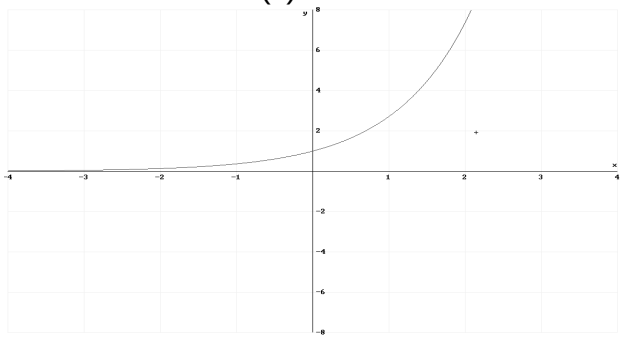
f(x) = $\frac{1}{x}$ (Hyperbel)



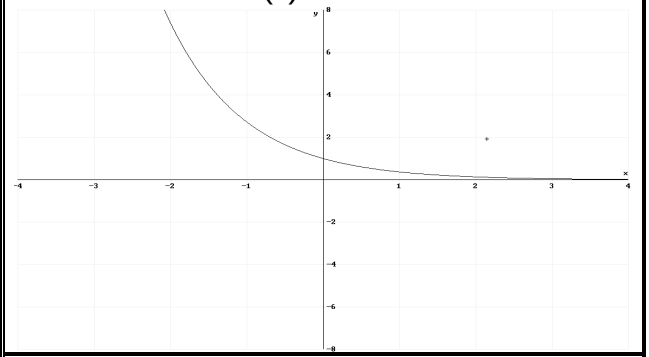
f(x) = $\frac{1}{x^2}$



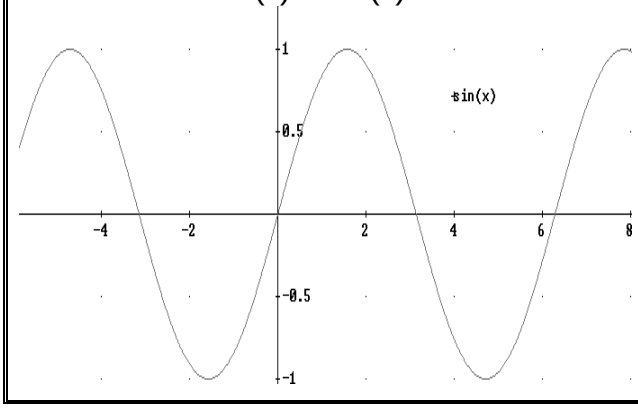
f(x) = $e^x$



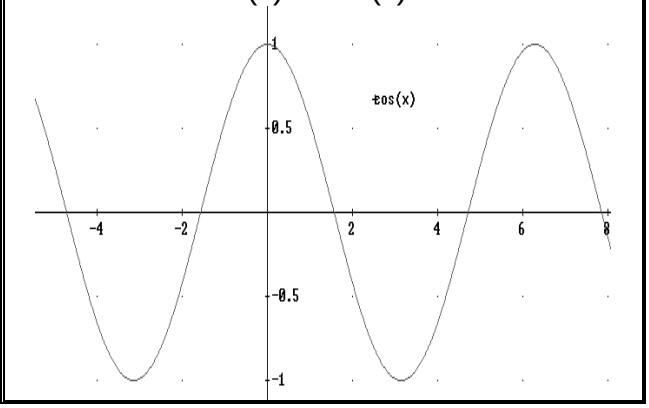
f(x) = $e^{-x}$



f(x) = sin(x)




f(x) = cos(x)




Last Update: 05.10.02